EKONOMICKÁ FAKULTA TU v Košiciach

Ekonomická fakulta
Technická univerzita v Košiciach

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Eco-efficiency and eco-productivity change over time in a multisectoral economic system

Mikulas Luptacik [†,+]

[+] *University of Economics Bratislava*
[†] *Institute for Industrial Research (IWI)*
[¥] *Vienna University of Economics and Business (WU)*

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Recessions are easily recognizable from a decrease in GDP. What really should interest us, however, is the difference between the potential of an economy and its actual performance.
(J. Stiglitz, 2002)

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1. Introduction

- Two strands of productivity analysis
  - neoclassical growth accounting
  - frontier approach (DEA)

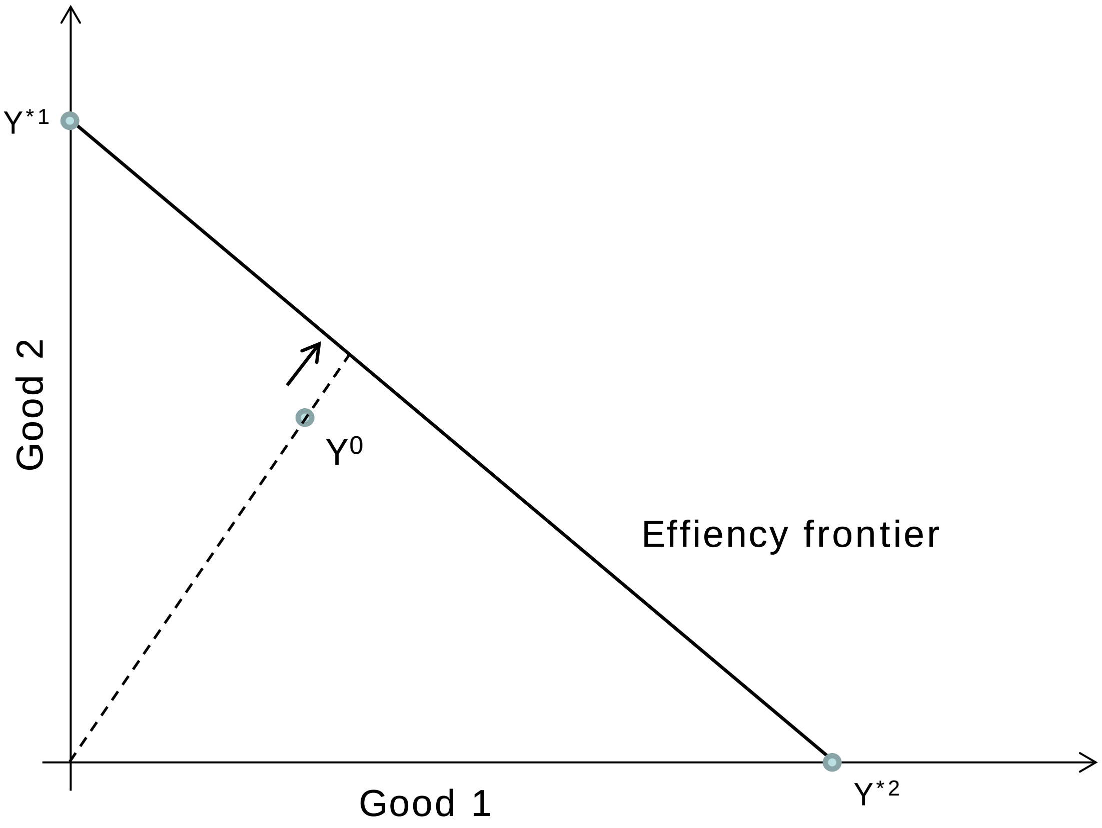
- „Neoclassical growth accounting imputes productivity growth to factors, but cannot distinguish a movement towards the frontier and a movement of the frontier. This is the contribution of the frontier approach, which, however, is not capable of imputing value to factor inputs“ (Thijs ten Raa – Pierre Mohnen, (2002) ➔ a synthesis of both approaches)

Procedure

The procedure consists of three steps:

1. **Generate the production possibility set**: Each good output is maximized subject to restraints on the production of other outputs, given environmental standards and available inputs (multi-objective optimization problem).
2. **Measure distance** of actual economy to the production frontier (with a DEA-model).
3. **Eco-efficiency change over time** based on Luenberger Indicator.

Production possibility set of the economy



Research Questions

Eco-productivity change:

- How to measure it?
- Where does it come from?
  Eco-efficiency change or eco-technical change?
- What are the main drivers?
  Is it output growth, input-saving or environmental-saving?
- How much do individual good outputs (agriculture, manufacturing, services, ...), bad outputs (air pollution, ...) and primary factors (capital, labor, ...) contribute?

2. Production possibility set of the economy

Leontief's input-output model

Economy with ***n* sectors**; Each sector produces a single homogeneous good, x_j. The j-th sector, in order to produce 1 unit, must use a_{kj} units from sector k. Furthermore, each sector sells some of its output to other sectors (intermediate output) and some of its output to consumers (net output, or final demand). Call final demand in the *j-th* sector y_j. Then we might write

$$x_j = a_{j1}x_1 + a_{j2}x_2 + ... + a_{jn}x_n + y_j$$

or total output equals intermediate demand plus final demand. If we let A be the indecomposable matrix of input coefficients a_{kj}, x be the vector of total (gross) output, and y be the vector of final demand/net output, then our expression for the economy becomes

$$x = Ax + y.$$

For given final demand the gross output must at least cover the intermediate output and final demand which can be written as

$$x \geq Ax + y \qquad \text{or} \qquad (I - A)x \geq y \qquad (1a)$$

The economy uses ***m* primary factors**. Moreover, the j-th sector, in order to produce 1 unit, must use b_{ij} units of the i-th primary factor. Then we might write

$$b_{i1}x_1 + b_{i2}x_2 + ... + b_{in}x_n = z_i$$

where b_{ij} the requirement of the j-th sector on the i-th primary factor and z_i the endowment of the i-th primary factor. Let B be the matrix of primary factor coefficients b_{ij} and z be the vector of total factor endowments. Then **the sum of primary factors used by all sectors can not exceed the total endowments in the economy**:

$$Bx \leq z \qquad (1b)$$

- To work out the efficiency measures and to derive the output potential of an economy with n outputs we face in principle a multi objective optimization problem. In many cases such problems are reduced to a single objective optimization problem by suitable aggregation (e.g. ten Raa (1995, 2005) uses world market prices for the n commodities employed in his model to reduce the optimization of n outputs to that of a single sum of values of the net products).

- Pursuing the multiple objective approach we propose to solve the following optimisation model where each net output y is maximised subject to restraints on the availability of inputs z^0:

$$\operatorname*{Max}_{x} \; y \qquad s.t.$$

$$(2) \qquad (I - A)x - y \geq 0$$

$$Bx \leq z^0$$

$$x, y \geq 0$$

- We use the notation “Max” for a vector optimization problem and “max” for a single objective problem.

- We solve, thus, n single objective problems where final demand for each commodity is maximized, i.e.

- (3) max y_j ($j = 1,\dots, n$)

- subject to the constraints in (2). For each of the n solutions of (3) denote the (also n-dimensional) solution vector x^*_j ($j = 1,\dots,n$) representing the gross productions of commoditites.

- Alternatively, for a given level of final demand y^0 the use of inputs z is minimized:

$$(4) \qquad \min_x z \quad s.t.$$

$$(I - A)x \geq y^0$$

$$Bx - z \leq 0$$

$$x, z \geq 0$$

- In this case, therefore, m single objective problems are solved

(5) min z_i (i = 1,..., m)

- subject to the constraints in (4). The m solution vectors x^*_i (i = 1,..., m) describe the gross production values of commodities for given final demand y^0 under the individual minimization of the primary factors i = 1,...,m.

- These sets of values of both problems defined above are arranged column-wise in a pay-off matrix with the optimal values appearing in the main diagonal while the off-diagonal elements provide the levels of other sector net-outputs and inputs compatible with the individually optimized ones. The payoff matrix of dimension (n+m x n+m) is written

$$P = \left[\begin{array}{cccc|ccc} \mathrm{y}^{*1} & \mathrm{y}^{*2} & \cdots & \mathrm{y}^{*\mathrm{n}} & \mathrm{y}^{0} + s_y^1 & \cdots & \mathrm{y}^{0} + s_y^m \\ \mathrm{z}^{0} - s_z^1 & \mathrm{z}^{0} - s_z^2 & \cdots & \mathrm{z}^{0} - s_z^n & \mathrm{z}^{*1} & \cdots & \mathrm{z}^{*\mathrm{m}} \end{array}\right] \equiv \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}$$

where s_y is the vector of the slack variables of the n outputs and s_z is the vector of the m input slacks.

- Each of the points in the payoff-matrix P is constructed independently of the other points but taking account of the entire systems relations. Knowing the efficient frontier we can estimate the efficiency of the actual economy. Each of the columns of the pay-off matrix can be seen as a virtual decision making unit with different input and output characteristics which all are using the same production technique. The real economy as given by actual output and input data defines a new decision making unit whose distance to the frontier can be estimated.

- Each of the columns of the pay-off matrix P can be seen as a virtual decision making unit for the DEA model which we use for measuring the efficiency of the economy given by the actual output and input data (y^0, z^0) in the following input oriented DEA problem

(6)

$$\min_{\mu} \theta \qquad s.t.$$

$$P_1\mu \geq y^0$$

$$-P_2\mu + \theta z^0 \geq 0$$

$$\mu \geq 0, \theta \geq 0$$

- where P_1 is the output matrix and P_2 the input matrix

3. The relationship between the DEA model and the LP-Leontief Model

- In the spirit of ten Raa (1995, 2005) and Debreu (1951) the Leontief-model (1) can be formulated as an optimization problem in the following way: minimize the use of primary inputs for given levels of final demand.

(7)

$$\min_x \gamma \qquad s.t.$$

$$(I - A)x \geq y^0$$

$$-Bx + \gamma z^0 \geq 0$$

$$x, \gamma \geq 0$$

- **Proposition 1:**
  - The efficiency score θ of DEA problem (6) is exactly equal to the radial efficiency measure γ of LP-model (7).
  - The dual solution of model (7) coincides with the solution of the DEA multiplier problem which is the dual of problem (6).
  - Proof: see Luptacik-Böhm 2010 p.613-614

For Pareto-Koopmans efficient solution we replace the radial model (6) by the following additive model:

$$\rho = \min_{\mu, s^-} 1 - (1/m) \sum_{i=1}^{m} \frac{s_i^-}{z_i^0} \quad \text{s.t.} \tag{8}$$

$$P_1 \mu \qquad \geq y^0$$

$$-P_2 \mu - \mathrm{s}^- = -\mathrm{z}^0$$

$$\mu, s^- \geq 0$$

where P_1 is the output matrix and P_2 the input matrix from the pay-off matrix.

In the spirit of ten Raa (1995, 2005) and Debreu (1951) the Leontief-model can be formulated as an optimization problem in the following way:

$$\omega = \min_{x,s^-} 1 - (1/m) \sum_{i=1}^{m} \frac{\hat{s}_i^-}{z_i^0} \quad \text{s.t.}$$

$$(I - A)x \geq y^0 \qquad (9)$$

$$-Bx - \hat{s}^- = -z^0$$

$$x, \hat{s}^- \geq 0$$

Proposition 2: The efficiency score ρ of DEA problem (8) is exactly equal to the efficiency measure ϖ of LP-model (9).
The dual solution of model (9) coincides with the solution of the DEA multiplier problem which is the dual of problem (8).

4. Extension to the augmented Leontief model

- The well known augmented Leontief model (Leontief, 1970) is written as

(10)
$$\begin{bmatrix} I - A_{11} & -A_{12} \\ -A_{21} & I - A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} y_1 \\ -y_2 \end{bmatrix}$$

$$\begin{bmatrix} B_1 & B_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = z$$

- Formulating the Leontief model as an LP-problem by minimising primary inputs for given levels of final demand y_1^0 and environmental standards y_2^0 we get

$$\min_{x} \gamma \qquad s.t.$$

(11)

$$(I - A_{11})x_1 - A_{12}x_2 \geq y_1^0$$

$$-A_{21}x_1 + (I - A_{22})x_2 \geq -y_2^0$$

$$-B_1x_1 - B_2x_2 + \gamma z \geq 0$$

$$x_1, x_2, \gamma \geq 0$$

- In analogy to section 2 we formulate the multiobjective optimization problem as follows

$$
\begin{aligned}
&Max \quad y \\
&(I - A)x - y \geq 0 \\
&Bx \leq z^0 \\
&x, y \geq 0
\end{aligned} \tag{12}
$$

where

$$
A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad y = \begin{bmatrix} y_1 \\ -y_2 \end{bmatrix}, \quad B = \begin{bmatrix} B_1 & B_2 \end{bmatrix}
$$

- We again solve n single objective problems maximizing final demand for each commodity separately:

(13) $\max \; y_1^j \quad (j = 1, \ldots, n)$

- s.t. the constraints in (12). Minimization of net pollution under the constraints (12) yields the trivial solution where all variables are zero. One could also formulate models which minimize pollution under the constraint that a given amount of output is obtained.

- Alternatively for given final demand y_1^0 and environmental standard y_2^0 (the tolerated level of net-pollution) the inputs z are minimized.

(14)

$$
\begin{aligned}
&Min \quad z \\
&(I - A)x \geq y^0 \\
&Bx - z \leq 0 \\
&x, y \geq 0
\end{aligned}
$$

- Solving the m separate single objective problems

(15) $\min z_i \quad (i = 1, \ldots, m)$

- we can derive the payoff matrix of dimension (n+k+m) x (n+m) for the augmented model partitioned in the following way

$$\begin{bmatrix} Q \\ Z \end{bmatrix} = \left[\begin{array}{ccc|ccc} y_1^{*1} & \cdots & y_1^{*n} & y_1^0 + s_{y_1}^1 & \cdots & y_1^0 + s_{y_1}^m \\ \hline y_2^1 & \cdots & y_2^n & y_2^0 - s_{y_2}^1 & \cdots & y_2^0 - s_{y_2}^m \\ \hline z^0 - s_z^1 & \cdots & z^0 - s_z^n & z^{*1} & \cdots & z^{*m} \end{array}\right] \equiv \begin{bmatrix} Q_1 \\ Q_2 \\ \hline Z \end{bmatrix}$$

- The DEA model related to the optimisation problem (11) is now

$$\min_{\mu} \theta \qquad s.t.$$

(16)
$$Q_1\mu \geq y_1^0$$

$$-Q_2\mu + y_2^0 \geq 0$$

$$-Z\mu + \theta z^0 \geq 0$$

$$\mu \geq 0, \theta \geq 0$$

- **Proposition 3:**

- The dual solution of model (11) coincides with the solution of the DEA multiplier problem (which is the dual of problem (16)).

- The efficiency score θ of DEA problem (16) is exactly equal to the radial efficiency measure γ of LP-model (11).

Production possibility set of the economy (1)

- Formulating the **Leontief model as an LP-problem** by minimizing primary inputs and maximizing final demand y_1^0 for given environmental standards y_2^0 we get

$$\max_x \delta \quad s.t.$$

$$\begin{aligned}
-\delta y_1^0 + (I - A_{11})x_1 - A_{12}x_2 &\geq y_1^0 \\
-A_{21}x_1 + (I - A_{22})x_2 &\geq -y_2^0 \\
\delta z^0 + B_1 x_1 + B_2 x_2 &\leq z^0
\end{aligned}$$

$$x_1, x_2 \geq 0, \delta \ldots \textit{free}$$

Production possibility set of the economy (2)

- The DEA model related to the optimization problem (3) is now

$$\max_{\mu} \beta \qquad s.t.$$

$$-\beta y_1^0 + Q_1\mu \geq y_1^0$$

$$Q_2\mu \leq y_2^0$$

$$\beta z^0 + Z\mu \leq z^0$$

$$\mu \geq 0, \beta \ldots \mathit{free}$$

Eco-productivity change of the economy over time (1)

The procedure shown above can be applied for inter-temporal analysis. For this purpose the well known **Luenberger-indicator** can be adopted.

$$L\left(z_t^0, y_{1,t}^0, y_{2,t}^0; z_{t+1}^0, y_{1,t+1}^0, y_{2,t+1}^0\right) = \frac{1}{2}\Big[\Big(\rho_{t+1}\left(z_t^0, y_{1,t}^0, y_{2,t}^0\right) - \rho_{t+1}\left(z_{t+1}^0, y_{1,t+1}^0, y_{2,t+1}^0\right)\Big) + \Big(\rho_t\left(z_t^0, y_{1,t}^0, y_{2,t}^0\right) - \rho_t\left(z_{t+1}^0, y_{1,t+1}^0, y_{2,t+1}^0\right)\Big)\Big]$$

where subscript t denotes time period and ρ distance functions.

The four distance function values (two single period for t and $t+1$ and two mixed-period distance functions) can be estimated by solving the **DEA model (8)** for the respective time period. For each DEA model a separate output matrix P_1 and a separate input matrix P_2 have to be constructed by solving the LPs (5) and (7). Consequently, these two models as well as model (8) have to be used four times.

Eco-productivity change of the economy over time (2)

Total Factor Productivity change obtained from the Luenberger indicator can be decomposed into a component of efficiency change (catch-up) and technology change (frontier shift), like for any other Luenberger indicators.

Efficiency change:

$$EFFCH\left(z_t^0, y_{1,t}^0, y_{2,t}^0; z_{t+1}^0, y_{1,t+1}^0, y_{2,t+1}^0\right) = \rho_t\left(z_t^0, y_{1,t}^0, y_{2,t}^0\right) - \rho_{t+1}\left(z_{t+1}^0, y_{1,t+1}^0, y_{2,t+1}^0\right)$$

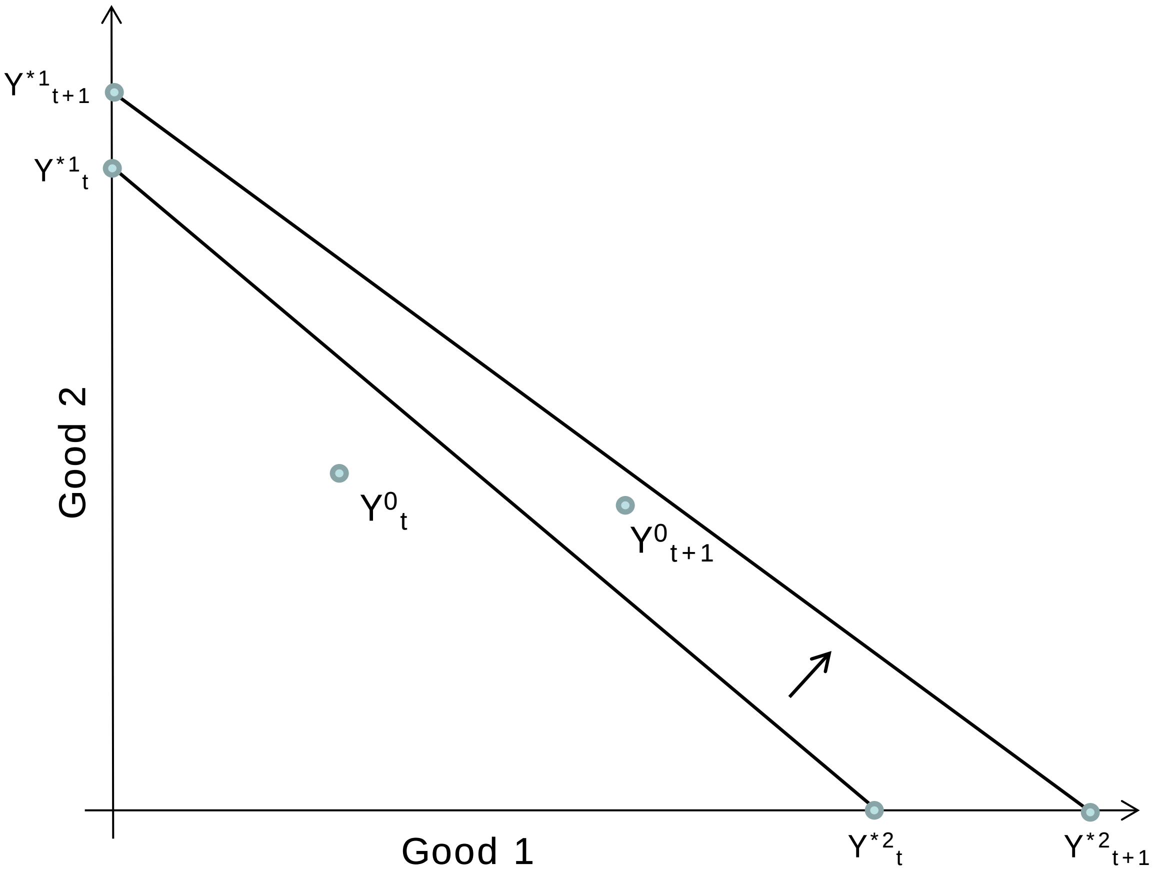
Technology change:

$$TECHCH\left(z_t^0, y_{1,t}^0, y_{2,t}^0; z_{t+1}^0, y_{1,t+1}^0, y_{2,t+1}^0\right) = \frac{1}{2}\Big[\Big(\rho_{t+1}\left(z_{t+1}^0, y_{1,t+1}^0, y_{2,t+1}^0\right) - \rho_t\left(z_{t+1}^0, y_{1,t+1}^0, y_{2,t+1}^0\right)\Big) + \Big(\rho_{t+1}\left(z_t^0, y_{1,t}^0, y_{2,t}^0\right) - \rho_t\left(z_t^0, y_{1,t}^0, y_{2,t}^0\right)\Big)\Big]$$

Productivity change:

$$L\left(z_t^0, y_{1,t}^0, y_{2,t}^0; z_{t+1}^0, y_{1,t+1}^0, y_{2,t+1}^0\right) = EFFCH\left(z_t^0, y_{1,t}^0, y_{2,t}^0; z_{t+1}^0, y_{1,t+1}^0, y_{2,t+1}^0\right) + TECHCH\left(z_t^0, y_{1,t}^0, y_{2,t}^0; z_{t+1}^0, y_{1,t+1}^0, y_{2,t+1}^0\right)$$

Eco-productivity change of the economy over time (3)



Eco-productivity change of the economy over time (4)

The proposed method attributes the use of individual inputs and the final demand of individual commodities to productivity change and its components. **Efficiency change**:

$$EFFCH\left(z_t^0, y_{1,t}^0, y_{2,t}^0; z_{t+1}^0, y_{1,t+1}^0, y_{2,t+1}^0\right) = \rho_t\left(z_t^0, y_{1,t}^0, y_{2,t}^0\right) - \rho_{t+1}\left(z_{t+1}^0, y_{1,t+1}^0, y_{2,t+1}^0\right) =$$

$$= \sum_{j=1}^{n} u_{1,j;t,t} y_{1,j;t}^0 - \sum_{j=1}^{n} u_{1,j;t+1,t+1} y_{1,j;t+1}^0 +$$

$$+ \sum_{j=1}^{n} u_{2,j;t,t} y_{2,j;t}^0 - \sum_{j=1}^{n} u_{2,j;t+1,t+1} y_{2,j;t+1}^0 + \sum_{i=1}^{m} v_{i;t,t} z_{i;t}^0 - \sum_{i=1}^{m} v_{i;t+1,t+1} z_{i;t+1}^0$$

Contribution of the *i*-th input: $v_{i;t,t} z_{i;t}^0 - v_{i;t+1,t+1} z_{i;t+1}^0$

Contribution of the *j*-th commodity: $u_{1,j;t,t} y_{1,j;t}^0 - u_{1,j;t+1,t+1} y_{1,j;t+1}^0$

Contribution of the *k*-th pollutant: $u_{2,j;t,t} y_{2,j;t}^0 - u_{2,j;t+1,t+1} y_{2,j;t+1}^0$

Eco-productivity change of the economy over time (5)

Technical change:

$$TECHCH\left(z_t^0, y_t^0, z_{t+1}^0, y_{t+1}^0\right) = \frac{1}{2}\left[\left(\rho_{t+1}\left(z_{t+1}^0, y_{t+1}^0\right) - \rho_t\left(z_{t+1}^0, y_{t+1}^0\right)\right) + \left(\rho_{t+1}\left(z_t^0, y_t^0\right) - \rho_t\left(z_t^0, y_t^0\right)\right)\right] =$$

$$= \frac{1}{2}\left[\sum_{j=1}^{n} u_{1,j;t+1,t+1} y_{1,j;t+1}^0 - \sum_{j=1}^{n} u_{1,j;t,t+1} y_{1,j;t+1}^0 + \sum_{j=1}^{n} u_{1,j;t+1,t} y_{1,j,t}^0 - \sum_{j=1}^{n} u_{1,j;t,t} y_{1,j;t}^0 + \right.$$

$$+ \sum_{j=1}^{n} u_{2,j;t+1,t+1} y_{2,j;t+1}^0 - \sum_{j=1}^{n} u_{2,j;t,t+1} y_{2,j;t+1}^0 + \sum_{j=1}^{n} u_{2,j;t+1,t} y_{2,j,t}^0 - \sum_{j=1}^{n} u_{2,j;t,t} y_{2,j;t}^0 +$$

$$\left. + \sum_{i=1}^{m} v_{i;t+1,t+1} z_{i;t+1}^0 - \sum_{i=1}^{m} v_{i;t,t+1} z_{i;t+1}^0 + \sum_{i=1}^{m} v_{i;t+1,t} z_{i;t}^0 - \sum_{i=1}^{m} v_{i;t,t} z_{i;t}^0 \right]$$

Eco-productivity change of the economy over time (6)

Contribution of the *i*-th input:

$$\frac{1}{2}\left(v_{i;t+1,t+1}z^{0}_{i;t+1} - v_{i;t,t+1}z^{0}_{i,t+1} + v_{i;t+1,t}z^{0}_{i;t} - v_{i;t,t}z^{0}_{i;t}\right)$$

Contribution of the *j*-th commodity:

$$\frac{1}{2}\left(u_{1,j;t+1,t+1}y^{0}_{1,j;t+1} - u_{1,j;t,t+1}y^{0}_{1,j;t+1} + u_{1,j;t+1,t}y^{0}_{1,j;t} - u_{1,j;t,t}y^{0}_{1,j;t}\right)$$

Contribution of the *k*-th pollutant:

$$\frac{1}{2}\left(u_{2,j;t+1,t+1}y^{0}_{2,j;t+1} - u_{2,j;t,t+1}y^{0}_{2,j;t+1} + u_{2,j;t+1,t}y^{0}_{2,j;t} - u_{2,j;t,t}y^{0}_{2,j;t}\right)$$

Eco-productivity change of the economy over time (7)

Total Factor Productivity change:

$$L\left(z_t^0, y_t^0, z_{t+1}^0, y_{t+1}^0\right) = \frac{1}{2}\left[\left(\rho_{t+1}\left(z_t^0, y_t^0\right) - \rho_{t+1}\left(z_{t+1}^0, y_{t+1}^0\right)\right) + \left(\rho_t\left(z_t^0, y_t^0\right) - \rho_t\left(z_{t+1}^0, y_{t+1}^0\right)\right)\right] =$$

$$= \frac{1}{2}\left[\sum_{j=1}^{n} u_{j;t+1,t}\, y_{j;t}^0 - \sum_{j=1}^{n} u_{j;t+1,t+1}\, y_{j;t+1}^0 + \sum_{j=1}^{n} u_{j;t,t}\, y_{j;t}^0 - \sum_{j=1}^{n} u_{j;t,t+1}\, y_{j;t+1}^0 - \right.$$

$$\left. + \sum_{i=1}^{m} v_{i;t+1,t}\, z_{i;t}^0 - \sum_{i=1}^{m} v_{i;t+1,t+1}\, z_{i;t+1}^0 + \sum_{i=1}^{m} v_{i;t,t}\, z_{i;t}^0 - \sum_{i=1}^{m} v_{i;t,t+1}\, z_{i;t+1}^0 \right]$$

Eco-productivity change of the economy over time (8)

Contribution of the *i*-th input:

$$\frac{1}{2}\left(v_{i;t+1,t} z^0_{i;t} - v_{i,t+1,t+1} z^0_{i;t+1} + v_{i,t,t} z^0_{i;t} - v_{i,t,t+1} z^0_{i;t+1}\right)$$

Contribution of the *j*-th commodity:

$$\frac{1}{2}\left(u_{1,j;t+1,t} y^0_{1,j;t} - u_{1,j;t+1,t+1} y^0_{1,j;t+1} + u_{1,j;t,t} y^0_{1,j;t} - u_{1,j;t,t+1} y^0_{1,j;t+1}\right)$$

Contribution of the *k*-th pollutant:

$$\frac{1}{2}\left(u_{2,j;t+1,t} y^0_{2,j;t} - u_{2,j;t+1,t+1} y^0_{2,j;t+1} + u_{2,j;t,t} y^0_{2,j;t} - u_{2,j;t,t+1} y^0_{2,j;t+1}\right)$$

Proposition 4: *The total contribution of the primary inputs* z^0 *is equal to the total contribution of all commodities (good outputs)* y_1^0. *This holds for PRODCH as well as for both components EFFCH and TECHCH.*

We show this for *EFFCH*. For *PRODCH* and *TECHCH* the relationship can be shown in an analogue way.

$$\begin{aligned}
EFFCH &= \rho_t\left(z_t^0, y_{1,t}^0, y_{2,t}^0\right) - \rho_{t+1}\left(z_{t+1}^0, y_{1,t+1}^0, y_{2,t+1}^0\right) = \\
&= \left(u_{1;t,t}y_{1,t}^0 + u_{2;t,t}y_{2,t}^0 + v_{t,t}z_t^0\right) - \left(u_{1;t+1,t+1}y_{1,t+1}^0 + u_{2;t+1,t+1}y_{2,t+1}^0 + v_{t+1,t+1}z_{t+1}^0\right) = \\
&= v_{t,t}z_t^0 - v_{t+1,t+1}z_{t+1}^0 + u_{1;t,t}y_{1,t}^0 - u_{1;t+1,t+1}y_{1,t+1}^0 + u_{2;t,t}y_{2,t}^0 - u_{2;t+1,t+1}y_{2,t+1}^0
\end{aligned}$$

$v_{t,t}z_t^0 - v_{t+1,t+1}z_{t+1}^0$ is the total contribution of all primary inputs and $u_{1;t,t}y_{1;t}^0 - u_{1;t+1,t+1}y_{1;t+1}^0$ the total contribution of all commodities and $u_{2;t,t}y_{2;t}^0 - u_{2;t+1,t+1}y_{2;t+1}^0$ the total contribution of all pollutants. It has to be shown that $v_{t,t}z_t^0 - v_{t+1,t+1}z_{t+1}^0 = u_{1;t,t}y_{1;t}^0 - u_{1;t+1,t+1}y_{1;t+1}^0$. After a short transformation of (27) we obtain $-u_{1;t,t}y_{1,t}^0 + v_{t,t}z_t^0 = -u_{1;t+1,t+1}y_{1,t+1}^0 + v_{t+1,t+1}z_{t+1}^0$. From (12) we know that $-u_{t,t}y_{1,t}^0 + v_{t,t}z_t^0 = 1$ and $-u_{t+1,t+1}y_{1,t+1}^0 + v_{t+1,t+1}z_{t+1}^0 = 1$ which complete the proof.

Empirical Application (1)

Country: Austrian economy

Observation period: 1995 to 2007

Input-Output Tables: aggregated to 18 commodity sectors, based on domestic use tables

Final demand: 18 commodities

Primary factors: low-skilled, medium-skilled und high-skilled labour
Capital stock (all assets)

Pollution: emissions to the air

Pollution abatement: expenditure for climate protection and pollution control

environmental standards: in 1995: 90% of cross emissions of 1995 and in 2007: 70% of cross emissions of 1995

Data sources: Statistics Austria (national accounts, integrated NAMEA (National Accounting Matrix including Environmental Accounts)), WIOD database

Empirical Application (2)

Descriptive statistics of **primary factors**:

	used	endow-ment	ratio used to endowment
		1995	
High-skilled labour (in millions hours)	741	767	0.97
Medium-skilled labour (in mill. hours)	3,974	4,219	0.94
Low-skilled labour (in millions hours)	1,435	1,792	0.80
Capital, all assets (in bill. EUR)	597	708	0.84
		2007	
High-skilled labour (in millions hours)	1,231	1,298	0.95
Medium-skilled labour (in mill. hours)	4,305	4,687	0.92
Low-skilled labour (in millions hours)	1,255	1,421	0.88
Capital, all assets (in Bill. EUR)	811	915	0.89

Empirical Application (3)

Descriptive statistics of **final demand** (selected commodities):

	1995	2007	growth
	in Bill. EUR		in percent
Primary sector (1 commodity)	1.15	1.64	42.48
Secondary sector (13 commodities)	84.34	131.28	55.66
Tertiary sector (4 commodities)	132.65	182.33	37.45
Total (all 18 commodities)	**218.14**	**315.25**	**44.51**
	in Mio. tons		in percent
Pollution	**44.66**	**34.73**	**-22.22**

Empirical Application (4)

Primary factor requirement (B-Matrix)
(How much resources are required to produce one unit of gross output?
How much resources are used to reduce one unit of emissions?):

1995

	Mean of 18 commodities	pollution abatement
High-skilled	1.47	0.05
Medium-skilled	13.57	0.08
Low-skilled	6.29	0.03
Capital total	1.35	0.79

2007

	Mean of 18 commodities	Pollution abatement
High-skilled	1.78	0.13
Medium-skilled	8.38	0.21
Low-skilled	2.81	0.08
Capital total	1.09	0.25

Emission coefficient (A_{21}-Matrix)
(How much gasses are emitted per unit of gross output?):

	1995	2007
Emission (in tons per 1,000 EUR production)	0.152	0.138

Empirical Application (5)

Results for single period DEA (model 8) and Leontief model (model 3) for 1995 and 2007:

	Eco-in-efficiency score	Shadow prices				
		Low-skilled labour	Medium-skilled labour	High-skilled labour	shadow pr. capital	shadow pr. pollution
1995						
DEA model	**0.016**	0	0	0.00066	0	0.00015
Leontief model	**0.016**	0	0	0.00066	0	0.00015
2007						
DEA model	**0.026**	0	0	0.00039	0	0.00004
Leontief model	**0.026**	0	0	0.00039	0	0.00004

Empirical Application (6)

Results of Luenberger indicator and components, 1995 to 2007:

	Eco-in-efficiency in 1995	Eco-in-efficiency in 2007	Mixed period 1995 to 2007	Mixed period 2007 to 1995	Eco-efficiency change	Eco-technical change	Eco-TFP change
DEA model	0.016	0.026	-0.174	-0.070	**-0.010**	**0.057**	**0.047**
Leontief model	0.016	0.026	-0.174	-0.070			

Efficiency change = 0.016 – 0.026 = -0.010

Empirical Application (7)

	Eco-efficiency change	Eco-technical change	Eco-TFP change
Primary sector (1 commodity)	0.003	0.009	0.012
Secondary sector (13 commodities)	0.017	0.051	0.068
Tertiary sector (4 commodities)	-0.028	-0.029	-0.056
pollution	**0.005**	**-0.006**	**-0.001**
High-skilled labour	-0.008	0.236	0.229
Medium-skilled labour	0	0	0
Low-killed labour	0	-0.205	-0.205
Capital	0	0	0
Total	**-0.010**	**0.057**	**0.047**

		Eco-efficiency change (*EFFCH*)	Eco-technical change (*TECHCH*)	Eco-productivity change (*PRODCH*)
Final demand	Products of agriculture, hunting, forestry and fishing	0.0033	0.0085	0.0118
	Mining	0.0004	0.0001	0.0005
	Food, beverages and tobacco	-0.0009	0.0230	0.0222
	Textiles and leather	-0.0008	0.0003	-0.0005
	Wood and products of wood	0.0030	0.0042	0.0072
	Paper and printed matter	0.0008	0.0022	0.0030
	Chemical and refined petroleum products	0.0006	0.0021	0.0026
	Other non-metallic mineral products	-0.0012	0.0007	-0.0004
	Basic metals	0.0010	0.0037	0.0047
	Machinery and equipment	0.0039	0.0087	0.0126
	Motor vehicles and transport equipment	0.0034	0.0039	0.0074
	Other manufactured goods	0.0012	0.0033	0.0044
	Electrical Energy	0.0000	0.0005	0.0005
	Construction Work	0.0053	-0.0013	0.0041
	Land transport services	-0.0024	0.0017	-0.0007
	Water transport services	-0.0001	0.0000	0.0000
	Air transport services	0.0005	0.0009	0.0014
	Other services and public administration	-0.0256	-0.0313	-0.0569
pollution	Air emissions	0.0054	-0.0055	-0.0001
primary inputs	low-skilled labour	0	-0.2049	-0.2049
	medium-skilled labour	0	0	0
	high-skilled labour	-0.0075	0.2362	0.2287
	capital	0	0	0
		-0.0096	**0.0570**	**0.0474**

Conclusions

The construction of the efficiency frontier permits an **assessment with respect to the own potential of an economy** defined by the given technology (even in the case of multiple outputs and inputs) **without the need to compare it with other economies** possessing possibly different technologies and obvious mutual interdependencies due to international trade.

Due to our results **the relative merits of both approaches (frontier approach and growth accounting)** can be used.

For inter-temporal comparisons of **Eco-productivity change** the **movement of the economy towards the frontier (Eco-efficiency change)** and **its shift (Eco-technical change)** can be obtained by using the DEA formulation.

Working paper is available at:

http://nhf.euba.sk/katedry/katedra-hospodarskej-politiky/working-papers